

Cosmic background radiation at 200 MHz

Determination of the Allan-variance for CALLISTO

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Created 2. 11. 2003

Abstract. This paper examines the possibility of measuring the step of 0.02 Kelvin in the spectrum at 200 MHz left by the reionisation epoch with the newly built spectrometer CALLISTO. To obtain the information on the resolution of CALLISTO the Allan-variance is computed.

Key words. reionisation, Allan-variance, CALLISTO

1. Introduction

As proposed by P. A. Shaver et al. (1999), the reionisation of the universe is expected to have left a signal in the form of a sharp step in the spectrum of the sky. This feature should be present in the radio sky at 70-240 MHz due to redshifted H I 21-cm line emission. This paper shall examine the possibility of measuring this step in the spectrum of approximately 0.02 K with the new spectrometer CALLISTO.

2. CALLISTO spectrometer

The CALLISTO (Compound Astronomical Low-cost Low-frequency Instrument for Spectroscopy and Transportable Observatory) spectrometer is a double heterodyne receiver newly built by the ETH Zürich Radio and Plasma Physics Group. It operates between 42 and 865 MHz using two modern, commercially available broadband cable-TV-tuners having a frequency resolution of 62.5 kHz. As the construction is still in progress, measurements were made on the development model, which was run with 200 channels at 400 Hz, i.e. every frequency is measured twice per second.

3. Data output and data processing

The data obtained from CALLISTO are spectrograms from 42 to 865 MHz for each tuner separately. The data are transferred via a RS232 cable to a computer and stored in a .raw-file. In order to process the data the .raw-file is

converted to two .fit-files; one containing the intensity information (tuner 0 + tuner 1) and the other the polarisation information (tuner 0 - tuner 1). As for the following analysis the polarisation is of no importance the tuners were operated in the power-splitting mode processing the same data. For convenient data handling several IDL routines were written.

4. Data conversion problems

While analysing the plots created from the .fit-files as described above, it became clear that the data had been corrupted during the conversion from the .raw-files to the .fit-files. The most obvious sign for this corruption were missing digits in the histogram. Firstly, truncation problems of the spectrometer in the four settings 8, 8.5, 9 and 10 bits were suspected but ruled out by experiments. Secondly, it was thought that the unlogarithmisation of the data (see section 5.6), which indeed does result in missing digits, had already taken place during the conversion. Peter Messmer (1999), the author of the conversion-routines, however assured that during the conversion the data from the two tuners were simply added (intensity) or subtracted (polarisation).

Comparing the spectrograms and lightcurves made from the .fit-files and .raw-files revealed eventually a similarity in the macroscopic distribution of the data, but inequalities in the microscopic distribution (missing peaks etc.). As these problems could not be fixed in time, the IDL routines had to be changed in order to read .raw-files. In the end, IDL routines for processing either .fit-files or .raw-files were available; the conversion problems however could not be solved until the end of this work.

5. Allan-variance

5.1. Theory

The variance σ^2 of certain number of measurements is defined by

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2,$$

where \bar{x} is the arithmetic mean value.

The process of averaging several datapoints into a single datapoint is called *integration*. If the integration takes place in time, the corresponding averaging interval is called integration time.

With increasing integration time, bandwidth and number of measurements, the standard deviation should decrease following the radiometer equation:

$$\frac{\sigma_F}{F} = \frac{1}{\sqrt{\Delta\nu n \tau}} \quad (1)$$

where F is the intensity of the signal, $\Delta\nu$ the bandwidth, n the number of measurements and τ the integration time. The variance is therefore proportional to $\frac{1}{\tau}$. In reality however systematic errors cause the variance to increase again after a certain integration time. The Allan-variance $\sigma_A^2(\tau)$ thus has a minimum and an optimal integration time. The plot $\sigma_A^2(\tau)$ is sometimes called an Allan-plot.

In the following sections we compute the ALLAN-variance of CALLISTO in order to evaluate its capability of measuring the redshifted H I line.

5.2. Implementation in IDL

For the computation of the Allan variance I followed the suggestions of V. Ossenkopf (2003). The data is given as datapoints $c_i(t_k)$, where the index i denotes the channel and t_k gives the time of the spectral dump with index k . In order to obtain a dimensionless result, which can be easily compared to other spectrometers, one may normalize the data by the mean value:

$$n_i(t_k) = \frac{c_i(t_k)}{\langle c_i(t_k) \rangle_k}$$

However, for our purpose this is not useful, as we need to know, how small our minimal standard deviation in Kelvin is.

The computation of the Allan variance consists in principle of a convolution of the data $c_i(t_k)$ by a Haar wavelet

$$H_\tau = \begin{cases} \frac{1}{\tau} & \text{for } -\tau \leq t < 0 \\ -\frac{1}{\tau} & \text{for } 0 \leq t < \tau \\ 0 & \text{everywhere else} \end{cases}$$

and therefore the Allan variance becomes:

$$\sigma_{A,i}^2(\tau) = \left\langle (c_i(t_k) * H_\tau - \langle c_i(t_k) * H_\tau \rangle_k)^2 \right\rangle_k$$

The integration time τ should be chosen as an integer multiple of the step size $\Delta t = t_{k+1} - t_k$, i.e. $\tau = l \Delta t$ with

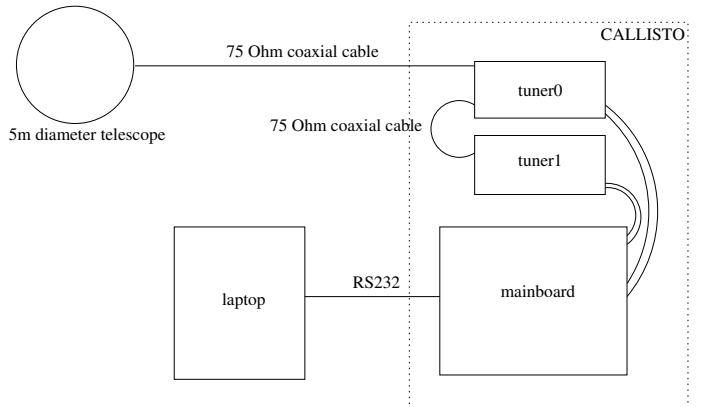


Fig. 1. Set up of the measurement with the 5m telescope.

$l = 1 \dots l_{max}$. l_{max} has to be chosen according to the fact that for reliable results the integration time should fall between one sixth and one third of the total length of the time series.

As the computation of the full convolution is very time consuming, the number of datapoints used for the computation is reduced by sampling the convolved function on a raster of $\frac{\tau}{2}$:

$$\sigma_{A,i}^2(\tau) = \left\langle (S_i(K) + T_i(K) - S_i(K+1) - T_i(K+1) - \langle S_i(K) + T_i(K) - S_i(K+1) - T_i(K+1) \rangle_K)^2 \right\rangle_K$$

where

$$S_i(K) = \frac{1}{l} \sum_{k=Kl}^{(K+1)l} c_i(t_k) \text{ and } T_i(K) = \frac{1}{l} \sum_{k=(K+\frac{1}{2})l}^{(K+\frac{3}{2})l} c_i(t_k)$$

The error estimate is thus obtained by

$$\delta\sigma_{A,i}^2(\tau) = \sqrt{\frac{1}{N} \left(\langle (c_i(t_k) * H_\tau - \langle c_i(t_k) * H_\tau \rangle_k)^4 \rangle_k - \frac{\langle (c_i(t_k) * H_\tau - \langle c_i(t_k) * H_\tau \rangle_k)^2 \rangle_k^2}{N} \right)}$$

with the total length of the time series equal to $N \tau$.

5.3. Set-up of the measurement at the 5 m-telescope in Bleien (Switzerland)

In order to obtain a matching wave resistance of 75 Ω throughout the system, the standard antenna of the 5 m telescope was removed and replaced with a commercial, logarithmic periodic, digital TV antenna with integrated 20 dB amplification and sensitive to the VHF/UHF range from 45 up to 870 MHz. As seen in figure 1 the signal is picked up with a parabolic reflector using the above mentioned antenna and transferred via a coaxial cable to CALLISTO where it is analysed by the two tuners in the power splitting mode. During normal operation the two tuners would be measuring the two different directions of the polarisation.

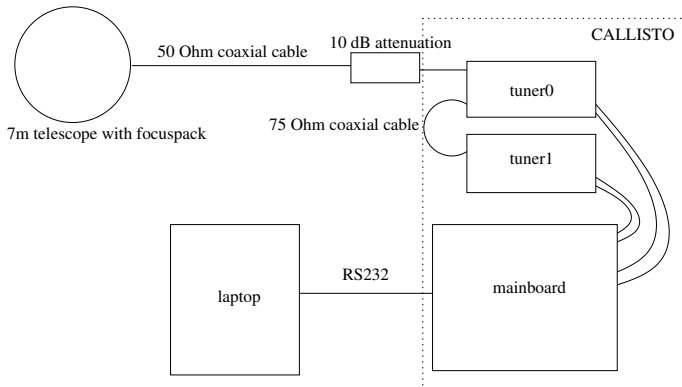


Fig. 2. Set up of the measurement with the 7m telescope.

5.4. Set-up of the measurement at the 7 m-telescope in Bleien (Switzerland)

So as to obtain the possibility of comparison measurements were also made using the 7 m telescope. The telescope is operated with a professional antenna between 100 and 4000 MHz and a focuspack. For the time of the measurement the spectrometer PHOENIX-II, which uses this telescope for sun observations, had to be unplugged. As seen in figure 2, the wave resistance was not matched throughout the system leading however to only small disturbances in the data. To lower the negative impact of a nearby pager transmitter (see section 6) the signal was attenuated by 10 dB.

5.5. Test for Gaussian distribution

As physical data is usually Gaussian like scattered around a mean value, plotting the histogram of a given lightcurve is a fast way for a quick examination, whether the system (spectrometer plus software) is behaving statistically correct or, whether the system produces some “false” values (see section 4 and Weinmann 2002). Considering figures 3 and 4, one can see that our distribution is indeed Gaussian and therefore the analysis in the following sections is valid.

5.6. Calibration

Since the Allan variance is needed in squared Kelvin a calibration has to be done using the sun as reference. For this purpose the telescope was first directed for several minutes at the sun and afterwards at the sky as seen in figure 5. This lightcurve consists of the following parts: 13:44 - 13:48 sun; 13:48 - 14:00 sky with disturbance picked up during the turning of the telescope; 14:00 - 14:06 forest. The trial of using the forest as reference failed completely for frequencies up to 300 MHz due to heavy disturbances emitted by transmitters for radio, pager, TV etc. while for higher frequencies the calibration might be possible but is very doubtful.

Before the calibration could be done the data had to be unlogarithmised by (1) converting the digits to Millivolts,

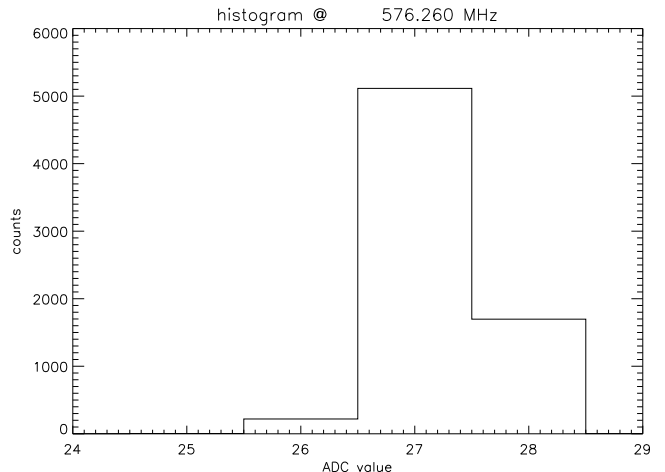


Fig. 3. Histogram for tuner 0 of a sky measurement on 10. 10. 2003 at 15:25.

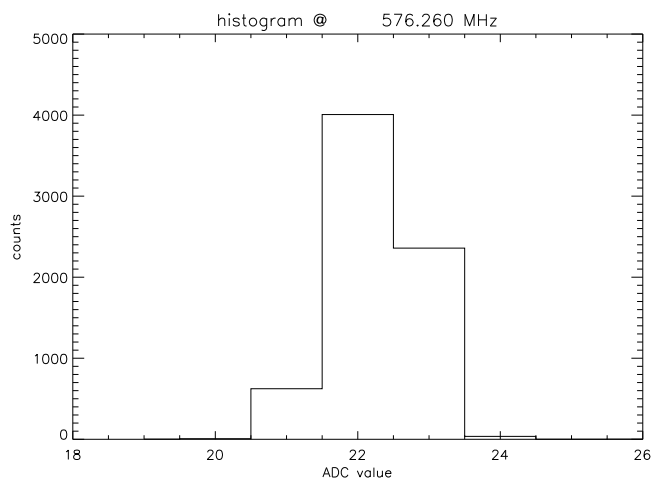


Fig. 4. Histogram for tuner 1 of a sky measurement on 10. 10. 2003 at 15:25.

(2) the Millivolts to decibels and (3) the decibels to linear power. For the first step the following equation was used:

$$data_{mV} = data_{digits} \frac{5000 \text{ mV}}{256}$$

where 5000 mV are the reference voltage and 256 the biggest integer in 8 bits.¹ The second step was done by knowing that at the operating point 50 mV correspond to 1 dB and the last step was easily handled by

$$data_{lp} = 10^{data_{dB}/10} .$$

Combining these three steps, one obtains:

$$data_{lp} = 10^{data_{digits} \frac{10}{256}}$$

Attention has to be paid to the fact that the data of the two tuners has to be unlogarithmised prior to the summation of the two.

¹ All the measurements were conducted in 8 bits.

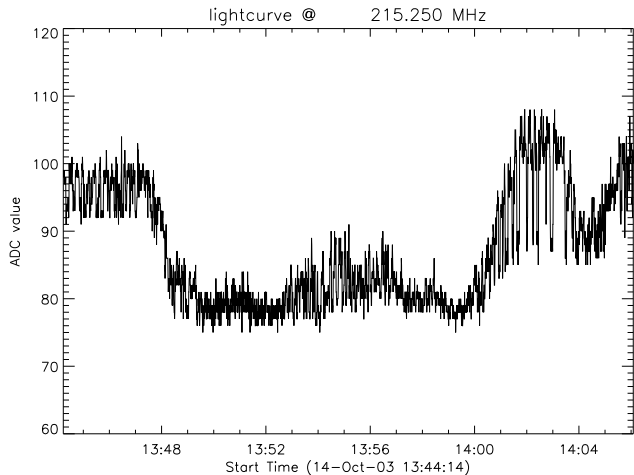


Fig. 5. Lightcurve used for calibration; sun, sky and heavily disturbed forest.

The first step of the calibration consisted of obtaining a relation between the digits and solar flux units (sfu)². For this reason the difference in digits between sun and sky $\Delta data_{digits}$ was computed. This difference is equal to the number in sfu published on the internet by the U.S. National Oceanic and Atmospheric Administration and the U.S. Air Force.³ Hence, a relation between digits and sfu is achieved. For the dependency in Kelvin we use the Rayleigh-Jeans law

$$data_K = data_{sfu} \frac{\lambda^2}{2k} \frac{G}{4\pi} 10^{-22},$$

where λ is the wavelength of the considered frequency, k the Boltzmann's constant and G the theoretical gain of the antenna calculated by

$$G_{dB} = 10 \log \eta + 20.4 \text{ dB} + 20(\log(D[m]) + \log(f[\text{GHz}]))$$

$$G = 10^{\frac{G_{dB}}{10}}$$

with $D[m]$ the diameter of the telescope in meter $f[\text{GHz}]$ the considered frequency in Gigahertz and an assumed efficiency factor η of 0.5. For a more precise result one would have to measure the gain of the antenna.

With these calculations we achieve a relative calibration which we need for the Allan variance. For an absolute calibration one would need the signal of two known sources such as T_0 or T_{exc} .

5.7. Results

In this section I will refer repeatedly to tuner 0 and tuner 1. Tuner 0 I relate to the tuner which writes into the first byte of the .raw-file; tuner 1 writes consequently into the second byte. Conducting a statistical analysis of the possibility of discerning two mean values with Gaussian distributions separated by 0.02 Kelvin one obtains that their

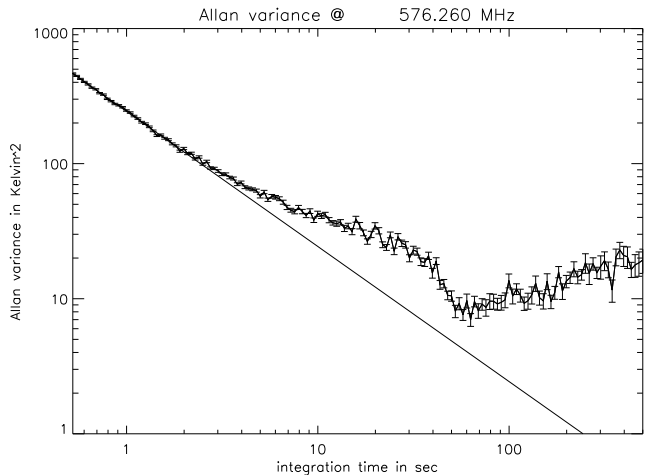


Fig. 6. Allan variance for tuner 0 of 60 minutes sky measurement with the 5m telescope; the solid straight line represents the radiometer equation (1).

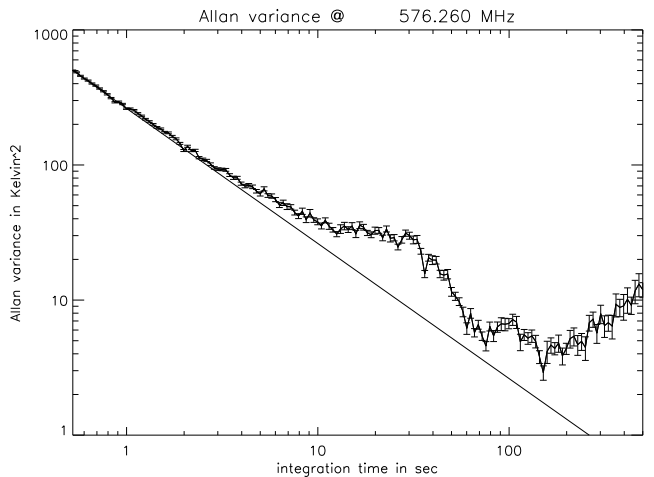


Fig. 7. Allan variance for tuner 1 of 60 minutes sky measurement with the 5m telescope; the solid straight line represents the radiometer equation (1).

standard deviations should not exceed approximately 0.02 Kelvin (see for example Kreyszig 1975); for the following results I will refer to this value.

The results extracted from the three figures 6, 7 and 8 are summarized in table 1. The reason for choosing this high frequency is due to the fact that all the lower frequencies are too heavily disturbed (see section 6).⁴ The data was taken on 10. October 2003 from 15:25 to 16:25 with the 5 m telescope pointing at the sky away from the sun and focuscode setting 6362 (left/right polarisation until 1 GHz). What one can easily see is that tuner 1 does have a lower variance than tuner 0, but is still lacking the necessary resolution of 0.02 K by a factor of 100. This difference in the sensitivity is explained by the fact that although these two tuners are the same model, there are

² 1 sfu = $10^{-22} \frac{W}{m^2 Hz}$; unit used in solar radio astronomy.

³ <http://www.sec.noaa.gov/ftpdir/lists/radio/> [2. 11. 2003]

⁴ As a reminder: The reionisation epoch leaves a signal at 200 MHz.

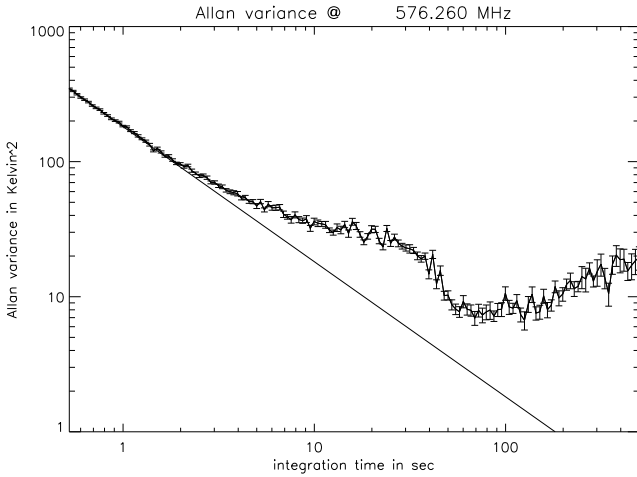


Fig. 8. Allan variance for tuner 0 + tuner 1 of 60 minutes sky measurement with the 5m telescope; the solid straight line represents the radiometer equation (1).

Table 1. Results for the sky measurement at 576.26 MHz with the 5 m telescope (see figures 6, 7 and 8).

	σ_A^2 at minimum	τ at minimum
tuner 0	$(8.4 \pm 0.9) \text{ K}^2$	$\approx 60 \text{ s}$
tuner 1	$(4.1 \pm 0.5) \text{ K}^2$	$\approx 150 \text{ s}$
tuner 0 + tuner 1	$(8.5 \pm 1.1) \text{ K}^2$	$\approx 120 \text{ s}$

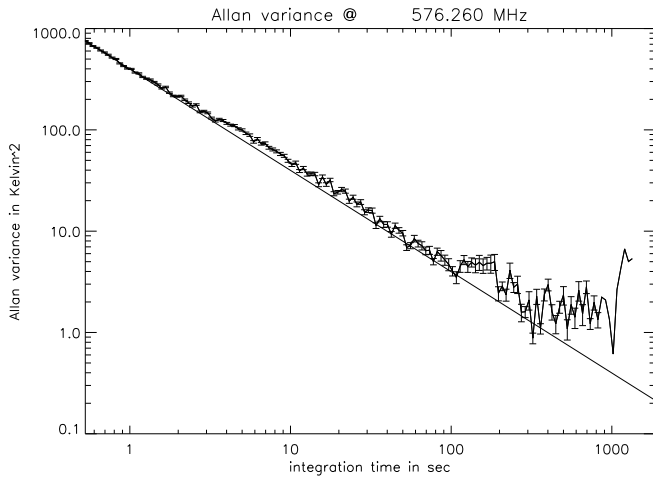


Fig. 9. Allan variance for tuner 0 of 69 minutes T_0 measurement with the 7m telescope; the solid straight line represents the radiometer equation (1).

always little variations during the manufacturing process resulting in slight differences in their behavior such as the sensitivity. Furthermore, it is to be noted that combining the two tuners does not improve the result. For the lowest possible variance one therefore has to identify the most sensitive tuner and measure with the selected tuner alone.

The second data collection was made on 14. October 2003 from 12:24 to 13:33 measuring T_0 of the focuspack at the 7 m telescope with the focuscode setting 5656 (50 Ω

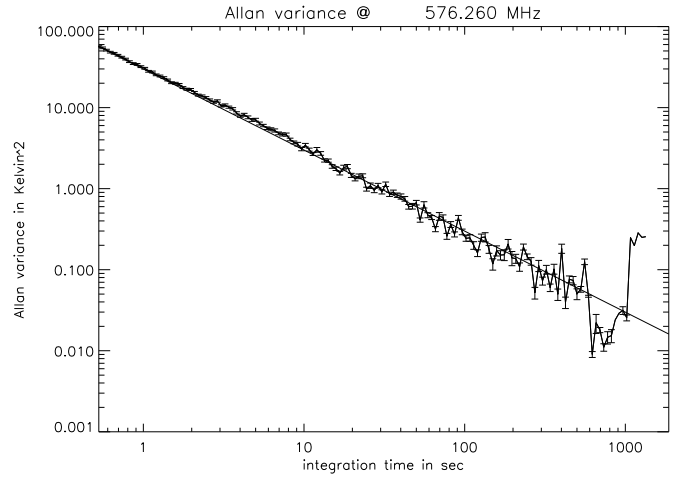


Fig. 10. Allan variance for tuner 1 of 69 minutes T_0 measurement with the 7m telescope; the solid straight line represents the radiometer equation (1).

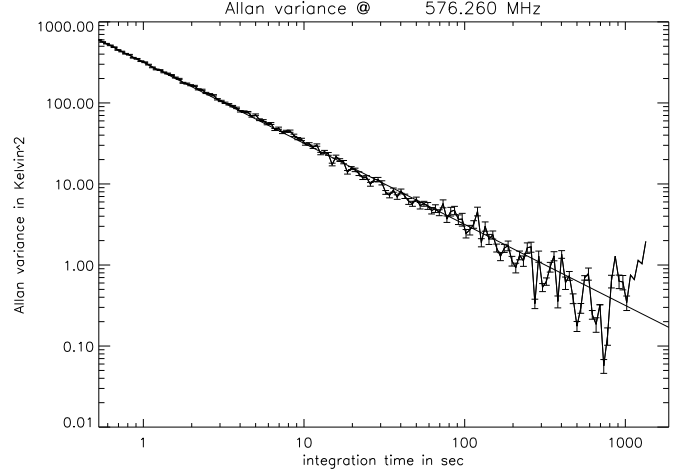


Fig. 11. Allan variance for tuner 0 + tuner 1 of 69 minutes T_0 measurement with the 7m telescope; the solid straight line represents the radiometer equation (1).

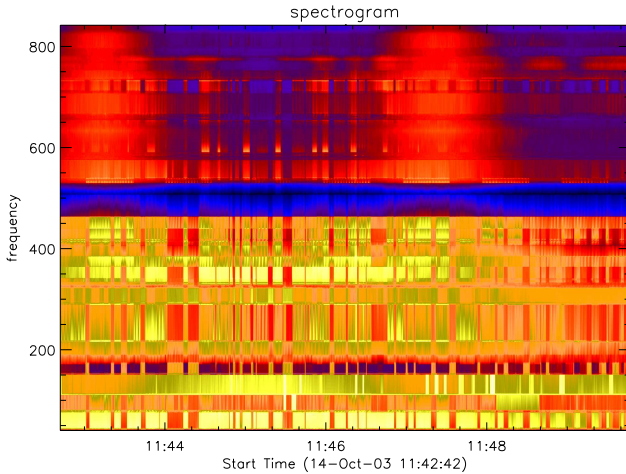
Table 2. Results for the T_0 measurement at 576.26 MHz with the 7 m telescope (see figures 9, 10 and 11).

	σ_A^2 at minimum	τ at minimum
tuner 0	$(1.8 \pm 0.2) \text{ K}^2$	$\approx 870 \text{ s}$
tuner 1	$(0.044 \pm 0.004) \text{ K}^2$	$\approx 620 \text{ s}$
tuner 0 + tuner 1	$(0.27 \pm 0.04) \text{ K}^2$	$\approx 730 \text{ s}$

termination until 1 GHz (right)). The telescope was in addition directed away from the sun to avoid possible influences on the data. For a direct comparison to the figures 6, 7 and 8 the same frequency was chosen (a lower frequency will be discussed below). The results from the three plots 9, 10 and 11 are presented in table 2; it is to be considered that due to the insufficient duration of the data collection approximately the last ten values in the diagrams 9, 10 and 11 are not to be regarded (see section 5.2).

Table 3. Results for the T_0 measurement at 215.25 MHz with the 7 m telescope.

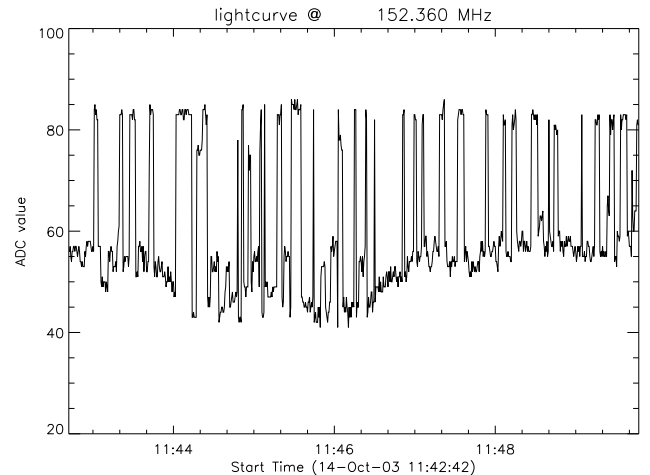
	σ_A^2 at minimum	τ at minimum
tuner 0	$(2.1 \pm 0.2) \text{ K}^2$	$\approx 420 \text{ s}$
tuner 1	$(0.16 \pm 0.03) \text{ K}^2$	$\approx 590 \text{ s}$
tuner 0 + tuner 1	$(0.62 \pm 0.08) \text{ K}^2$	$\approx 420 \text{ s}$

**Fig. 12.** Spectrogram taken with the 7m telescope showing two sun passages and heavy disturbances basically up to 800 MHz.

Evidently, the results in table 2 are much better than in table 1 and are backing the above statements that (1) tuner 0 is less sensitive than tuner 1 and (2) the combination of the tuners does not improve the sensitivity below the sensitivity of the better tuner. The resolution of tuner 1 is only a factor of 10 too big in order to detect the step of 0.02 Kelvin; although it is to say that this result is only partly significant for this comparison of needed resolution, because almost all outside disturbances are blocked out by measuring T_0 in the focuspack. A more realistic set up as with the 5 m telescope shows that outside disturbances play a major role leading to an insufficient resolution of about 2 orders of magnitudes. Effects of outside disturbances penetrating the shielded focuspack can even be seen during the T_0 measurements at lower frequencies, where the disturbances have their maximum, resulting in higher Allan-variances and lower integration times (see table 3).

6. Problems and suggestions

Obviously terrestrial disturbances do have the biggest negative impact on the sensitivity of the system. Near the telescope site in Bleien a pager and cellular phone transmitter is installed making observations in this frequency range practically impossible (see figures 12 and 13); but also devices like laptops emitting electromagnetic waves too close to the receiver produce unwanted effects. Biggest improvements can therefore be made by reducing these effects by

**Fig. 13.** Lightcurve extracted from the figure 12.

measuring at radio quiet zones⁵, shielding CALLISTO and removing discerned disturbances in the data.

More subtle are the disturbances originating from the preamplifier and CALLISTO (especially the tuners). Regarding the figures 9 and 10 for tuner 0 the limiting factor is the tuner itself as the tuner 1 is unaffected. For tuner 1 however it is unclear if the final limit will be caused by the amplifier or the tuner. Several tests will have to be conducted to examine these dependencies in more detail.

A formidable task will also be the constancy of temperature needed for the desired high precision measurements. So far only the room where CALLISTO was installed is regulated by an air conditioning system with an accuracy of 0.5 Kelvin. A system regulating the temperature for CALLISTO alone with higher accuracy would be worthwhile. Moreover, it would be desirable to being able to regulate the temperature at the antenna - in this context however a wish not feasible. After all the possible improvements by temperature regulation will have to be determined in experiments first.

Ingenious ways of data processing being able to extract the wanted information despite the disturbances may render some of the above mentioned improvements on the hardware side unnecessary.

7. Conclusion

Regarding the principal goal of measuring the step in the spectrum left by the reionisation epoch as introduced in section 1, I come to the conclusion that the detection of this 0.02 Kelvin trace is not a priori impossible. However, major improvements still have to be undertaken before reaching the needed resolution.

Acknowledgements. I would like to thank Hansueli Meyer and Christian Monstein for the technical support, Pascal Saint-Hilaire for the IDL crash course, Fabienne Carniello and

⁵ As already sought after by Christian Monstein: <http://www.astro.phys.ethz.ch/instrument/callisto/RFSPEC2/spectrum2.htm> [2. 11. 2003]

Christian Liechti for the correction and Arnold O. Benz for the theoretical explanations and the possibility to work on this project.

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