Noise|| Noise: Y-factor versus Signal-to-Noise Ratio

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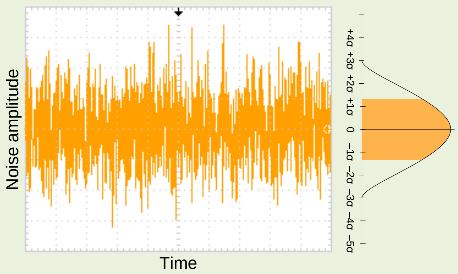
Often when people talk about noise, one is never sure what they are really talking about. Even scientists sometimes mix up the two noises, one based on the Y-factor and the other based on a comparison to the background noise.In this paper we explain the difference and use the observation of solar radio bursts to illustrate some practical calculations.

Keywords: Callisto, Y-factor, signal-to-noise ratio, SNR

Introduction

A radio telescope's sensitivity is a measure of the weakest celestial radio emission that can be detected with confidence. Sensitivity is directly related to the errors of measurement. The emissions ("signals") we measure usually appear as a small change in the receiver output as the radio source passes through the antenna beam or, as in the case of solar radio bursts, as an increase above the background noise level. The measurement errors and detection limits are determined by the fluctuations in the receiver output. Noise, which is inevitable, causes these fluctuations. There are many causes of noise and not all of them are completely random and follow a normal, or Gaussian, distribution (see sidebar **Gaussian Noise**); nevertheless, we can assume they are Gaussian to simplify the mathematics needed to analyze the radio telescope outputs.

Gaussian noise, also called *Additive White Gaussian Noise* (AWGN), is noise that has probability density function equal to a normal distribution, the familiar bell-shaped probability plot shown below. The area under the probability plot between two points is the probability that the amplitude will fall within those two points.



Noise || Noise

Noise has zero average amplitude around its mean value, but the mean value itself does not have to be zero. For example, if the output of a 5 Vdc power supply is closely examined, there will be in addition to ac ripple some random noise due to the semiconductors (shot noise) and resistors (Johnson noise) in the power supply. The average, or mean, value of this noise will be 5 V. The noise amplitude about the mean is described by its standard deviation, usually indicated by the symbol σ (sigma). In spreadsheet programs it is indicated by the function STDDEV. The root mean square (rms) amplitude of noise equals 1 standard deviation, or 1σ. If we make a large number of instantaneous noise amplitude measurements, approximately 68.3% will fall within ±1σ of the mean and about 99.7% will fall within ±3σ. The equation for σ is described in any statistical mathematics handbook (for example, see [Davenport]).

We can calculate the properties of emissions received by our radio telescopesin a couple ways. First, we can measure the ratio of the emission's peak noise power to the mean value of the background noise power. This is called the Y-factor. We also can measure the ratio of the received emissions noise power ("signal") to the statistical variations in the background noise amplitude. This is called the signal-to-noise ratio, SNR, or, because all signal measurements include noise, it is more accurately signal + noise-to-noise ratio. Y-factor is often confused with SNR, but the two measurements are not the same.

Y-Factor || Signal-to-Noise Ratio

As mentioned above, the Y-factor is the ratio of two noise power levels. In the two examples that follow, we measure one power at the peak of a received emission (*hot* measurement) and the other at the average value of the background noise (*cold* measurement). It should be noted that the hot measurement includes background noise.

Y is calculated from

$$Y = \frac{I1}{I0} \qquad [1]$$

where *I*0 is the average noise power or intensity and *I*1 is the noise at the peak of the output. Usually the Y-factor is expressed in dB, or

$$YdB = 10\log(Y) \qquad [2]$$

Y-factor is widely used in measuring the gain, noise temperature and noise factor of an amplifier. The Y-factor also is important in describing the radio telescope frontend, which is the subsystem that converts the incoming radiation into an electrical signal compatible with the detector system. The components influencing the Y-factor are the radio source's flux, antenna effective area, noise figure of the low noise amplifer (LNA) at the antenna and the level of background noise power. The background noise itself is a combination of the galactic background, including cosmic microwave background (CMB) radiation, internal noise of the first amplifier and spillover from the ground and a few other minor contributions.

Signal-to-noise ratio compares the peak intensity of the received emissions to the statistical characteristics of the background noise itself. Since the emissions received on Earth from most celestial radio sources are very weak, they are easily masked by the excursions of the background noise. If the rms of the noise is too high, a weak emission may be buried within it and undetectable. Therefore, the rms of the noise or some multiple of the rms indicates the detection probability. For example, an output due to the received emissions increasing > 2σ above the mean noise might be considered significant because the probability of such an output caused by ordinary noise is only 0.02. If we require higher confidence, the output must increase by a higher multiple. If the output increase is > 3σ, the probability that it is ordinary noise decreases to 0.001. Multiples of 2 to 5 are common.

The signal-to-noise ratio is defined as

$$snr = \frac{I1 - I0}{\sigma} \qquad [3]$$

where σ denotes the rms of the background noise level. Usually the SNR is expressed in dB, or

$$snrdB = 10\log(snr) \qquad [4]$$

Example 1

To illustratethe difference in signal-noise and background-noise, we use light-curve data from a solar radio burst in the VHF range; see figure 1. The burst's peak intensity occured near 129.4 MHz, so we can copy and paste the data at that frequency from the associated 2-dimensional spectrum (intensity, frequency and time) into a spreadsheet or other program and plot it; see figure 2. We need to make several calculations at the frequency in question to determine the Y-factor of the burst andbthe signal-to-noise ratio of the burst.

We used an 8-bit Callisto for our measurements. The measured intensities are proportional to the loading of Callisto's analog-digital converter (ADC), expressed in digits. For an 8-bit ADC the loading can vary from 0 to 255. In our example radio burst, the peak intensity I1 = 185 digits and the background-noise intensity I0 = 121 digits. In this case, the Y-factor is 1.52 (Eq. [1]) or 1.84 dB (Eq. [2]). Both values used to calculate the Y-factor have a common reference (zero ADC loading).

We can derive σ (rms noise or STDDEV) directly from the background noise by measuring its peak-peak power amplitude. We mentioned earlier that multiples of σ in the range 2 to 5 are common. If we choose 3σ for reliable detection, then the peak-peak noise is ± 3σ = 6σ. The minimum and maximum noise powers marked by blue dashed lines in figure 2 are designated s0 and s1. Their difference is set to 6σ, or

$$6\sigma = s1 - s0 \qquad [5]$$

and

$$\sigma = \frac{s1 - s0}{6}$$

In our example (figure 2), the noise is given by an upper value s1 = 128 digts and a lower value s0 = 123 digits. Therefore, the noise σ = (128 digits – 123 digits)/6 = 0.83digits. We can now calculate the SNR from Eq. [3], or snr = (185 digits – 121 digits) / 0.83 digits = 77.1 or 18.9 dB.

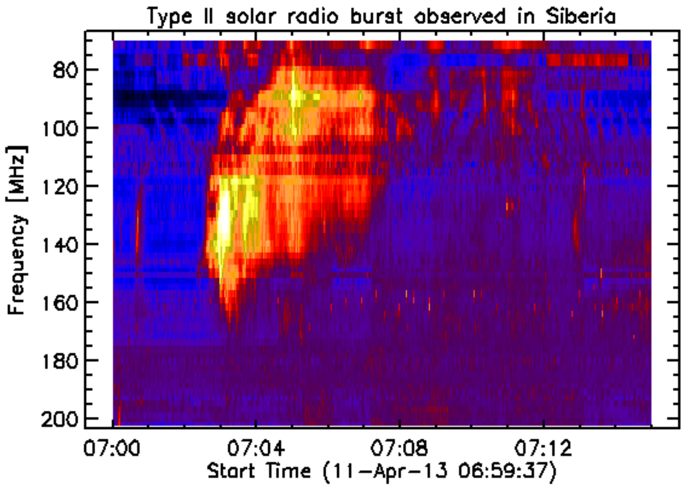


Figure 1~ Two-dimensional spectrum plot of a typical solar radio type II burst observed at Siberian Solar Radio Telescope SSRT in Badary near Baikal Lake, Russia. We use the data from this spectrum to calculate the Y-factor and SNR at the peak of the burst around 129.4 MHz.

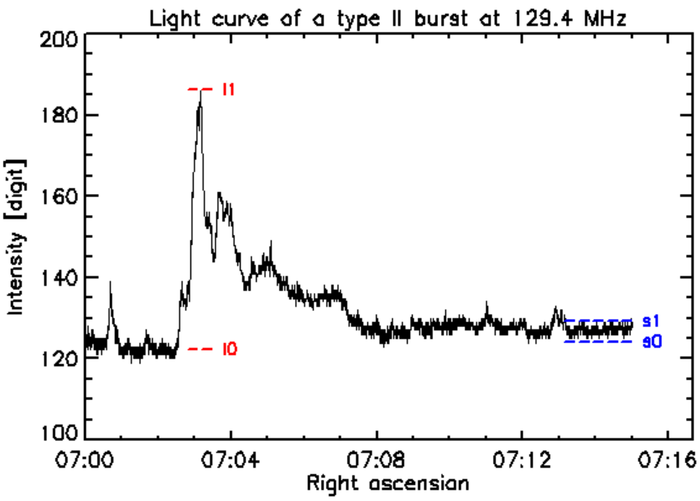


Figure 2 ~ Light-curve extracted from figure 1 at 129.4 MHz showing peak signal amplitude bove the background noise of the observed burst (difference between red dashed lines I1 and I0). I0 itself describes the background noise level.The blue dashed lines labeled s1 and s0 describes the 6 sigma (or 6 rms or 6 stdev) noise variance.

Conclusion

We tried to explain the difference between Y-factor and signal-to-noise ratio (SNR) which, in fact are totally different but colloquially treated as more or less the same. Be careful when discussing about noise, your discussion counterpart may talk about something else.

Links:

Callisto general information: http://www.e-callisto.org/

References and further reading

[Benz (2004)] Arnold O. Benz, Christian Monstein and Hansueli Meyer, CALLISTO, A New Concept for Solar Radio Spectrometers, Kluwer Academic Publishers, The Netherlands, 2004

[Davenport] William Davenport, Jr. and William Root, An Introduction to the Theory of Random Signals and Noise, IEEE Press, 1987