How did Kraus (1986) get the crazy factor 4/3?

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Abstract. On page 3-7 Kraus (1986) uses 4/3 in its solution to an exercise. He refers to chapter 6 but one sees nothing in chapter 6 that would explain the spooky factor 4/3. Here an appropriate theory is presented to understand how this factor was introduced in the early years of radio astronomy.

Key words. Directivity, beam angle, beam efficiency, pattern shape factor

1. Theory

Very often (Kraus, 1986) uses his crazy factor 4/3 to calculate directivity D, gain G, beam solid angle Ω_a or beam angles Θ respective Φ. First off all, lets write down the basic equation for further analysis

$$D = \frac{4\pi}{\Omega_a} = \frac{4\pi\ \epsilon_m}{k_p\ \Theta_{rad}\ \Phi_{rad}} = \frac{41'253\ \epsilon_m}{k_p\ \Theta_{deg}\ \Phi_{deg}} \tag{1}$$

where ϵ_m describes the beam efficiency factor. For practical use Kraus (1986) suggests $\epsilon_m = 0.75 \pm 0.15$. In European countries one is used to work with stray factor β_s instead of ϵ_m describing the influence of the side lobes where

$$\epsilon_m = 1 - \beta_s \tag{2}$$

Θ and Φ describe the half power beam width of the chosen antenna in two polarization directions. The middle part of Eq. 1 refers to angles given in radians while the right part refers to the alternative where degrees are used. The exotic factor $41'253$ can exactly be calculated by

$$41'253 = (180/\pi)^2\ 4\pi \tag{3}$$

The other factor k_p describes the so called pattern shape factor, where $k_p = 1.05 \pm 0.05$. *Remark: For amateur purposes k_p may be set to 1.0 with only minor influence to the end result.* If we now transpose Eq. 1 for antenna beam solid angle Ω_a we then get

$$\Omega_a = \frac{4\pi\ k_p\ \Theta_{rad}\ \Phi_{rad}}{4\pi\ \epsilon_m} = \frac{k_p}{\epsilon_m}\ \Theta_{rad}\ \Phi_{rad} \tag{4}$$

In reality ϵ_m can be evaluated by integrating a transit meridian scan of a astronomical radio source like the Sun or the Moon. In this case the main lobe and the sides lobes have to be evaluated separately. If we then put in the above mentioned values for ϵ_m and k_p we then get

$$\Omega_a = \frac{1.05}{0.75}\ \Theta_{rad}\ \Phi_{rad} = 1.4\ \Theta_{rad}\ \Phi_{rad} \tag{5}$$

which in first order is

$$\Omega_a \simeq \frac{4}{3}\ \Theta_{rad}\ \Phi_{rad} \tag{6}$$

where neither 4 nor 3 has any physical background. They are only simple numbers with absolutely no meaning.

2. Final result

I personally don't recommend that crazy factor 4/3 like in Eq. 6 because every antenna is an individual hardware object with its individual efficiency respective individual efficiency factor ϵ_m. Thus I strongly suggest to use Eq. 4 (right part) instead.

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References

John. D. Kraus, *Radio Astronomy 2nd edition*, Cygnus-Quasar Books 1986, p. 3-7 (example+solution), p. 6-6 Eq. 6-22.